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Petri Net Modelling of Spatial and Temporal Constraints in Track Allocation Problem at Railway Stations

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Abstract: The track allocation problem (TAP) at a multi-track, multi-platform mainline railway station is defined by the station track layout and service timetable, which implies combinations of spatial and temporal conflicts. Feasible solutions are available from either traditional planning or advanced intelligent searching methods and their evaluations with respect to operational requirements are essential for the operators. To facilitate thorough analysis, a timed Coloured Petri Nets (CPN) model is presented here to encapsulate the inter-relationships of the spatial and temporal constraints in the TAP.

Introduction: The track allocation problem (TAP) is a sub-problem of mainline railway service scheduling. With given track layout and service timetables at a multi-track, multi-platform station, a conflict-free path, composed of an arrival route and a departure route, is allocated to each train for traversing and/or stopping at the station. The essence of solving TAP is to avoid spatial and temporal conflicts due to track layout and timetable respectively while safety restrictions, service requirements for passengers boarding, alighting and transferring are also the constraints. Service disruptions are however inevitable in real-life operation. Robustness of a TAP solution is its capability to absorb disruptions and it is often introduced as buffer time between train services. If a number of feasible solutions to the TAP are available, it is essential to look for the one with the highest robustness while maximising the utilisation of the station capacity. To enable evaluation of the possible solutions to TAP, the spatial and temporal conflicts have to be modelled effectively and the possible buffer times are embodied in the model.

In this paper, a timed Coloured Petri Nets (CPN) approach is presented to model the relationships among service timetable, station track layout and buffer times. This model allows systematic representation of train movement at stations and hence simple analysis on the robustness of TAP solutions upon service disturbances.

Station track allocation: A mainline railway station is characterised by a number of entry and exit points, track sections and platforms. A train, when timetabled to pass through or stay at the station, has to be assigned a specific route for arrival and another for departure. TAP is to eliminate conflicts among the routes. Various approaches have been employed to attain solutions to TAP [1], [2]. In general, their objective is to maximise the number of trains to be routed, or to allocate priorities on chosen routes. However, the service quality of the resulting track allocation scheme has rarely been addressed. When robustness is one of the commonly used indicators of service quality, the feasibility solutions for TAP have to be evaluated properly on robustness.

Spatial and temporal conflicts: TAP is categorised as a resource allocation problem. With reference to a job shop scheduling problem, a temporal conflict occurs when two jobs must be processed simultaneously and it can be eliminated by assigning parallel machines to the jobs or introducing a buffer time between them. If two jobs need to use the same machine, it is a spatial conflict, and it is resolved by providing a buffer time to absorb the possible disturbances in real-time operation. Evidently, if it is possible to insert more buffer times between jobs, the schedule is more robust spatially and temporally. Resources are however limited, and the required buffer times are not necessarily available to avoid conflicts. As a feasible solution to TAP is embedded with certain buffer times, its robustness is implicit and the evaluation of the solution requires proper modelling of the spatial and temporal conflicts at the station.

An arrival or departure route of a train at a station is composed of sequential track sections, each section is occupied by no more than one train at any one time, or a spatial conflict occurs. An example of a portion of track layout at a station is illustrated in Fig.1 and it is composed of 10 sections, g1, and g2… g10 despite the simple track layout. From Fig.1, train t1 arrives at track 3 through route r1=<g1, g2, g6> train t2 on track 1 and train t3 on track 4 head for the exit through routes r2=<g7, g2, g1, g3>and r3=<g9, g4, g3>, respectively. Track sections g1 and g2 are the conflict areas for t1 and t2, and g3 for t2 and t3. On the other hand, trains t1 and t3 are free from spatial conflict.

A track section and a train can be regarded as a machine and a job in a job-shop problem. If the processing times of two jobs overlap, a temporal conflict occurs. The timings of the trains using the tracks 1, 2, 3 and 4 are listed in Fig.2. The time diagram of a train is divided into 3 parts. The left-hand triangle denotes the route setup time for the train to stop at or pass through the platform track while the right-hand triangle indicates the release time of the departure route when the train leaves. s(t1, r1) is the setup time for t1 to stop at track 3 through route r1 and s(t2, r2) is the release time for t2 to leave track 1 through route r2. They can be obtained through simple train movement simulator [3]. The rectangle in the middle is the station dwell time. If a(t1) and d(t1) are the scheduled arrival and departure times for t1, the dwell time is d(t1) - a(t1). Trains t1 and t3 are in temporal conflict, so are t3 and t4.

Buffer times: To avoid conflicts, buffer time is introduced between two trains which are scheduled to occupy the same track section and its amount reflects the capability to tolerate delays. In TAP, the station layout and service timetables are fixed, different solutions to TAP allow different amounts of buffer times to be inserted and hence their relationships with the track layout and timetables have to be included in the conflict model.

Timed Coloured Petri Nets modelling: Petri Nets is a powerful and flexible modelling technique for discrete event systems [4]. It is particularly suitable for modelling concurrent and asynchronous behaviours and it has found successful applications on job shop scheduling problem modelling [5]. Petri Nets has also been adopted to construct a large-scale railway infrastructure [6], in which the spatial characteristics of station layouts are modelled. For the modelling of TAP, conflicts, concurrency of train movement and buffer times exist and hence timed CPN approach is employed. The construction and validation of timed CPN model are now well supported by commercially available CPN software tools [7]. The CPN symbols adopted below follows the syntax of CPN ML language [7].

With the basic CPN model shown in Fig.3, a train and a free track section are denoted by the timed Colour Sets P and Q respectively. There are four places and two transitions in the model to represent the process of a train entering, staying and leaving a track section. The tokens in the places are ready to be consumed no sooner than the timestamp indicated (if shown) at the places. Arcs t1 and g1 denote a train and a track section; the output arc of transition ‘Occupy’ refers to the events that a train occupies the track section, and the value ‘90’ in the arc expression ‘(t1,g1)@+90’ represents the time-span (in seconds) of the event of occupation. The place ‘g1 busy’ shows the state of the track section being occupied by the train t1. The transition ‘Release’ denotes train departure from the track section which becomes ‘free’ afterwards and thus ready to be occupied by the subsequent train.

The first step of the CPN model is to represent the movements of individual trains over their designated routes in the basic model. The movements of trains t1 and t2 over routes r1 and r2 in Fig.1 are illustrated in Fig.4 (a) and (b) respectively. The transition ‘Enter’ is the event that train t1 starts entering the station via the entry and ‘Leave’ represents the event of train t2 starts departing from the station. As the track sections g1 and g2 are common to r1 and r2, the places ‘g1 busy’ and ‘g2 busy’ and the transitions ‘Release g1’ and ‘Release g2’ appear on both Fig.4 (a) and (b). With t1 and t2 running in opposite directions, they proceed through g1 and g2 in opposite sequences. The next step of the model is to combine the CPN diagrams through the interactions at the common track sections, in addition to constraints from the timetable.

If the timetable stipulates that t2 starts its departure 300s ahead of the entry of t1, the combined CPN diagram, as shown in Fig.5, incorporates the spatial and temporal constraints. The starting point is the combined place ‘Trains’ and the initial marking 1`t1@300+++1`t2@0 represents there are one t1 and one t2 in the place, and the scheduled starting time of t1 at 300s and that of t2 at 0s. The places ‘g1 busy’ and ‘g2 busy', and transitions 'Release g1' and 'Release g2' are now common to both t1 and t2. The interlocking relationships of the track sections reflect the spatial constraints. When timetable and run-times of trains through each track section (for simplicity, 50s for t1 for all track section; and 30s for t2) are given, the temporal constrains between places are taken in. A simple example is that the timestamp on the place 'g7 busy' indicates 30s for t2 to keep g7 from being released.

From the transition 'Release g1', the next event to be triggered (either the place 'g2 busy' or 'g3 busy') depends on the 'input' to the transition. To ensure an unambiguous model, a variable x is introduced in the inscription of the arc from 'g1 busy' to 'Release g1'. The Colour Set of x is P defined in Fig.3, representing the train at the place 'g1 busy'. In addition, the out-going arcs from 'Release g1' are also defined according to the value of x. The transition 'Release g2' is also specified similarly.

From Fig.5, t1 is allowed to start only when the scheduled time is reached and the route (g1, g2 & g6) is clear. The latter is now bound the movement of t2 which starts earlier. It is shown on Fig.5 that the transition 'Enter' is cleared to proceed when t2 has gone through g7, g2 and g1 and a total of 90s is elapsed. In other words, the buffer time is implicitly included in the model, i.e. 300s – 90s = 210s. Further, it is also possible to include service disturbances as additional places and transitions in the model.

Conclusions: Timetables and station layout impose spatial and temporal constraints on the TAP. A timed CPN model is proposed to encapsulate both constraints in a systematic depiction while the buffer time also is embedded in the model implicitly. This model enables proper evaluation of possible solutions for TAP with respect to robustness and thus service quality.

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Figure captions:

Fig. 1 Spatial conflict between trains

Fig. 2 Temporal conflict and buffer time between trains

Fig. 3 A timed CPN model of a train and a track circuit section

Fig.4 CPN models for trains t1 and t2

Fig. 5 Combined CPN model for trains t1 and t2

Figure 1

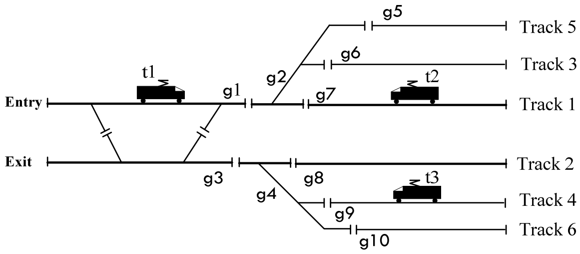


Figure 2

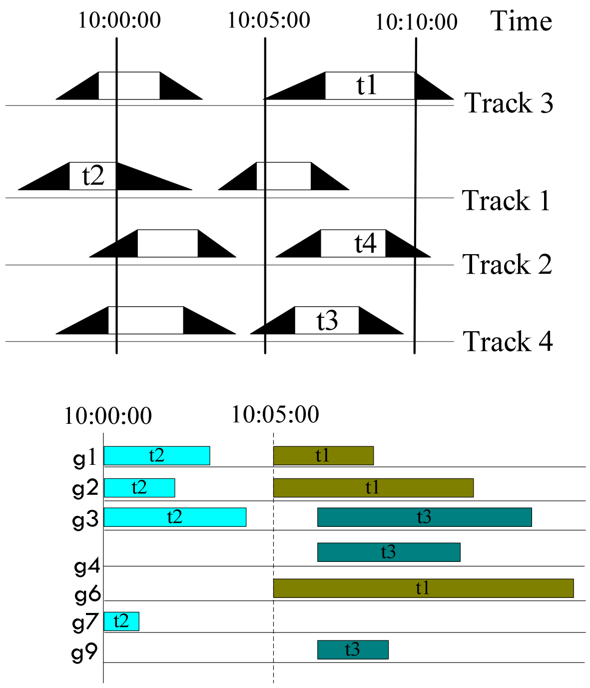


Figure 3

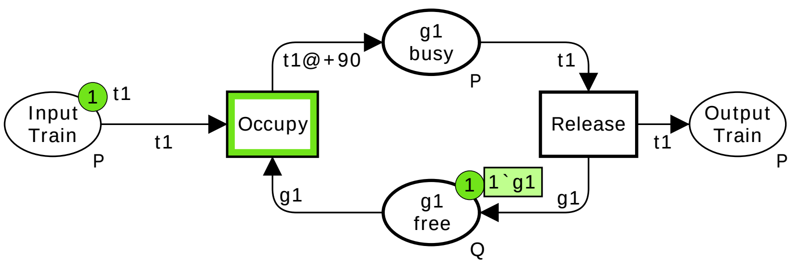
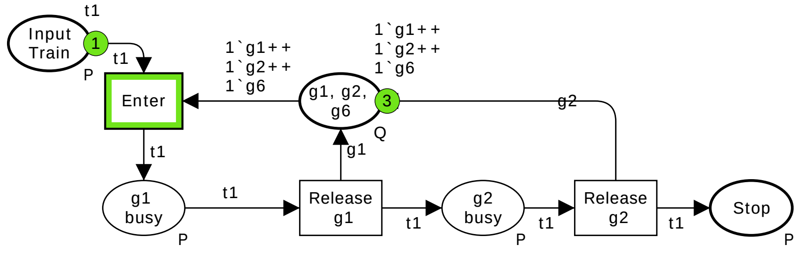


Figure 4

(a)


Input Train
t1
P
Enter
1`g1++
1`g2++
1`g6
g1, g2, g6
3
Q
g2
g1
g1 busy
Release g1
g2 busy
Release g2
Stop

(b)

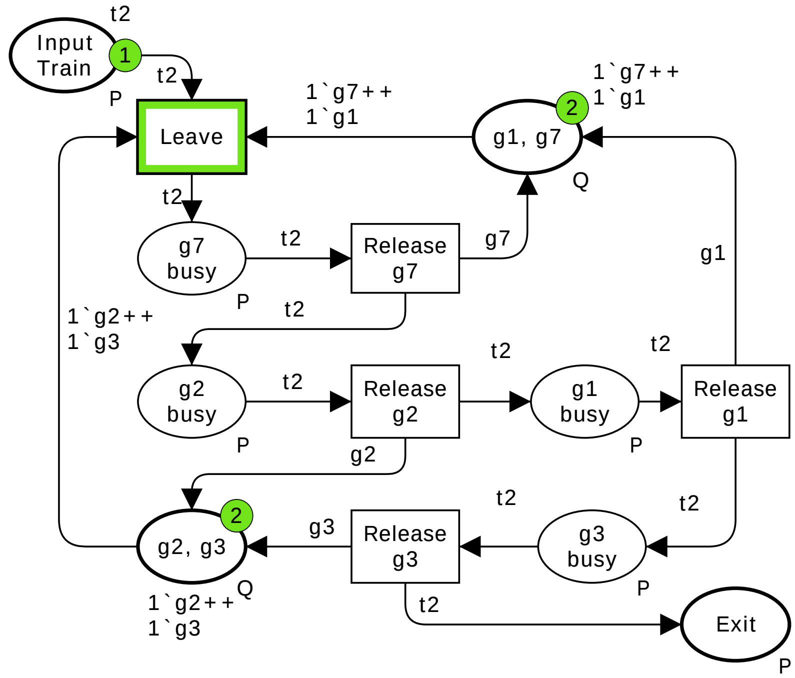
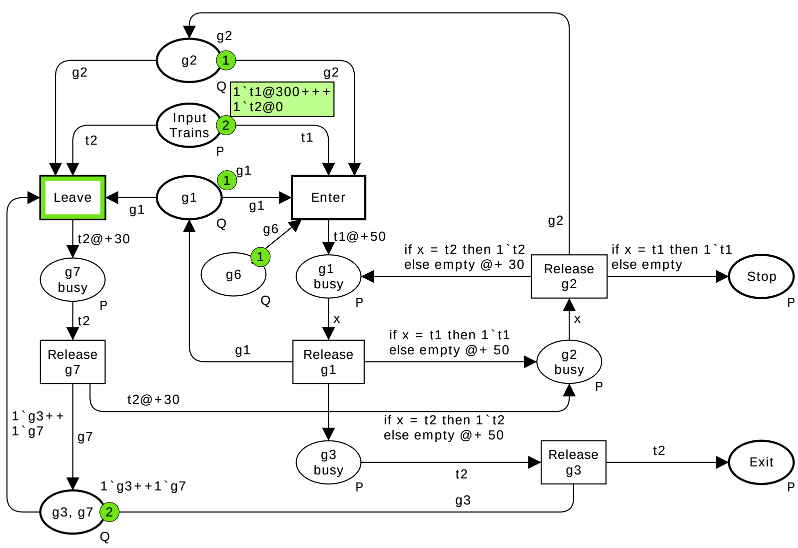

Input Train
t2
P
Leave
1`g7++
1`g1
g1, g7
Q
g7 busy
Release g7
g7
g1
1`g2++
1`g3
g2 busy
Release g2
g1 busy
Release g1
g2
g2, g3
g3
Release g3
g3 busy
Exit

Figure 5


g2
Q
1`t1@300+++
1`t2@0
Input Trains
t2
t1
P
Leave
g1
Enter
g6
t1@+50
t2@+30
g7 busy
g1 busy
if x = t2 then 1`t2
else empty @+ 30
Release g2
if x = t1 then 1`t1
else empty
Stop
Release g7
Release g1
x
if x = t1 then 1`t1
else empty @+ 50
g2 busy
1`g3++
1`g7
g7
if x = t2 then 1`t2
else empty @+ 50
g3 busy
Release g3
Exit
1`g3++1`g7
g3, g7
g3